Radiative corrections to the Higgs potential in the LH model

Antonio Dobado, Lourdes Tabares-Cheluci
Departamento de Física Teórica I,
Universidad Complutense de Madrid, E-28040 Madrid, Spain

Siannah Peñaranda
Departamento de Física Teórica, Universidad de Zaragoza, E-50009, Zaragoza, Spain

Javier Rodriguez-Laguna
Departamento de Matemáticas, Universidad Carlos III de Madrid, E-28911, Madrid, Spain

ABSTRACT

In this work we compute the radiative corrections to the Higgs mass and the Higgs quartic couplings coming from the Higgs sector itself and the scalar fields ϕ in the Littlest Higgs (LH) model. The restrictions that the new contributions set on the parameter space of the models are also discussed. Finally this work, together with our three previous papers, complete our program addressed to compute the relevant contributions to the Higgs low-energy effective potential in the LH model and the analysis of their phenomenological consequences.

1 Introduction

The discovery of a Higgs boson and the elucidation of the mechanism responsible for the electroweak symmetry breaking are some of the major goals of present and future searches in particle physics. Because of the precise data obtained for a long time to test the Standard Model (SM) of particle interactions, and the recent measurements of the W and the top masses at the Fermilab Tevatron [1], the SM has been confirmed as the right model describing the electroweak phenomena at the current experimental energy scale. However, the origin of the electroweak symmetry breaking, for which the Higgs boson is responsible in the SM, remains elusive. The quadratically divergent contributions to the Higgs mass and the electroweak precision observables imply different scales for physics beyond the SM, being the first one below 1 TeV and the second one above 10 TeV. This is the so called *little hierarchy problem*. As it is well known the mass of the Higgs boson receives one-loop corrections that are quadratic in the loop momenta. The largest contributions come from the top quark loop, with smaller corrections coming from loops of the electroweak gauge bosons and of the Higgs boson itself. Cancellations between the top sector and other sectors must occur in order to have the Higgs mass lighter than 200 GeV as expected from the electroweak precision test of the SM, which requires a fine-tuning of one part in 100. As this situation is quite unnatural various theories and models have been designed to solve this problem.

An interesting attempt to deal with it is the so called *Littlest Higgs model* (LH) [2], inspired in an old suggestion by Georgi and Pais [3], which tries to solve the little hierarchy problem by adding new particles with masses $O(\text{TeV})$ and symmetries which protect the Higgs mass from those dangerous quadratically divergent contributions (see [4] and [5] for reviews). These particles include the Goldstone bosons (GB) corresponding to a global spontaneous symmetry breaking (SSB) from the $SU(5)$ to the $SO(5)$ group, a new third generation vector quark called T and the gauge bosons corresponding to an additional gauge group which contains at least a $SU(2)_R$ and eventually a new hypercharge $U(1)$. In this case, and contrary to the supersymmetric theories, cancellation occurs between same-statistics particles. However, LH models typically leave uncanceled logarithmic divergencies which requires additional new contributions at some higher scale to preserve a small Higgs boson mass. Many of such models with different *theory space* have been constructed [2, 6], and electroweak precision constraints on various little Higgs models have been investigated by performing global fits to the precision data [7–11]. The existence of the different new states in these models could give rise to a very rich phenomenology, which could be probed at the CERN Large Hadron Collider (LHC) [12, 13].

Nevertheless, it is clear that any viable model has to fulfill the basic requirement of reproducing the SM model at low energies. In particular, from the LH model it is possible in principle to compute the Higgs low-energy effective potential and then, by comparing with the SM potential, to obtain their phenomenological consequences including new restrictions on the parameter space of the LH model itself. For example, one can obtain the one-loop contribution to the parameters of the standard Higgs potential,

$$V = -\mu^2 HH^\dagger + \lambda(HH^\dagger)^2; \tag{1.1}$$

where μ^2 and λ denote the well known Higgs mass and Higgs self-couplings parameters. Then it is possible to set restrictions over the LH parameters space by imposing the condition $\mu^2 = \lambda v^2$, where v is the SM vacuum expectation value ($H = (0, v)/\sqrt{2}$). The μ^2 sign and value are well known [2, 13], and effectively they are the right ones to produce the electroweak symmetry breaking, giving a Higgs mass $m_H^2 = 2\mu^2$. However, the full expression for the radiative corrections to λ has not been analyzed in detail so far. In principle both μ^2 and λ receive contributions from fermion, gauge boson and scalar loops, besides others that could come from the ultraviolet completion of the LH model. We have previously computed the contributions to the Higgs effective potential in the LH model coming from the fermion sector and the gauge boson sector [14, 15]. On the other hand, several relations for the threshold corrections to the λ parameter in the presence of a 10 TeV cut-off, depending on the UV-completion of the theory, have been reported (see, for example [17]). Besides, we have computed the effective potential for the doublet Higgs and the triplet ϕ [16], coming from the fermionic and gauge boson one-loop contributions and from the higher order effective operators needed for the ultraviolet completion of the model.

In [14] and [15] we computed and analyzed the fermion contributions to the low energy Higgs effective potential together with the effects of virtual heavy and electroweak gauge bosons present in the LH model. We have illustrated in these works the kind of constraints on the possible values of the LH parameters that can be set by requiring the complete LH effective potential to reproduce exactly the SM potential. The radiative corrections to λ, at the one-loop level, had not been previously computed. The computation of λ is important for several reasons: First, it must be positive, for the low energy effective action to make sense. In addition, from the effective potential (1.1), one gets the simple formula $m_H^2 = 2\lambda v^2$ or, equivalently, $\mu^2 = \lambda v^2$, where v is set by the experiment (for instance from the muon lifetime) to be $v \simeq 245$ GeV. In our phenomenological discussion in [14, 15] we have shown that the one-loop effective potential of the LH model cannot reproduce the SM potential with a low enough Higgs mass, $m_H^2 = 2\lambda v^2 = 2\mu^2$, in agrement with the present experimental constraints.

In order to solve this problem we computed in [16] the effective potential for the doublet Higgs and the triplet ϕ; coming from the fermionic and gauge boson one-loop contributions and also from the higher order effective operators, as defined in [12]. The relevant terms of this effective potential can be read as,

$$\begin{aligned} V_{eff}(H,\phi) &= -\mu_{fg}^2 HH^\dagger + \lambda_{fg}(HH^\dagger)^2 \\ &\quad + \lambda_{\phi^2} f^2 \mathrm{tr}(\phi\phi^\dagger) + i\lambda_{H^2\phi} f(H\phi^\dagger H^T - H^*\phi H^\dagger)\,, \end{aligned} \qquad (1.2)$$

where $\mu_{fg}^2 > 0$ and $\lambda_{fg} > 0$.

With this potential we studied the regions of the LH parameter space giving rise to the SM electroweak symmetry breaking. Although radiative corrections from fermion and gauge boson loops were discussed in [14, 15], the radiative contributions to λ_{ϕ^2} and $\lambda_{H^2\phi}$ have not been computed so far. New constraints over the LH parameter space emerge once we impose the new relation between coefficients of the effective Higgs potential namely; $v^2 = \mu_{fg}^2/\lambda_{fg} - \lambda_{H^2\phi}^2/\lambda_{\phi^2}$. In particular, the lowest value found for the μ parameter was 390 GeV [16], which implied a Higgs boson mass of about $m_H \simeq 550$ GeV, still not compatible with the present experimental constraints.

On the other hand it is well known that the radiative corrections coming from the Higgs itself and the ϕ fields could also provide relevant contributions to the effective potential. Thus the main goal of the present work is to check wether these corrections could really reduce the Higgs mass to solve the above mentioned problem, making the LH model compatible with the present phenomenology.

This work is organized as follows: In Section 2 we briefly explain the LH model. A summary on the SSB and the mass eigenstates is presented in Section 3. We set the notation in the two aforementioned sections. Section 4 is devoted to the computation of the radiative corrections contributions to the Higgs mass and quartic coupling coming from the scalar sector loops. In Section 5 we analyze the constraints that our computation establishes on the LH parameters and, finally, in Section 6 we present the conclusions. The expressions of the coefficients of the effective potential (1.2) coming from the radiative corrections and the effective operators are listed in the Appendix.

2 The model

The LH model is based on the assumption that there is a physical system with a global $SU(5)$ symmetry that is spontaneously broken to a $SO(5)$ symmetry at a high scale Λ through a vacuum expectation value ($v.e.v$) of order f. Thus, 14 Goldstone bosons (GB) are obtained as a consequence of this breaking. In this work we will consider two different versions of the LH model. In the first one the $SU(5)$ subgroup $[SU(2)\times U(1)]^2$ is gauged. We refer to this version as *Model I.* In the second one the gauge group is $[SU(2)^2\times U(1)]$ (*Model II*) [14, 15]. In both cases some of the GB acquire masses through radiative corrections coming from the gauge bosons and the t, b and T fermions loops.

The starting Lagrangian of the LH model is given by [2, 12, 13]:

$$L = L_\Sigma + L_{YK} \tag{2.1}$$

where L_Σ is the Non Linear Sigma Model (NLSM) lagrangian:

$$L_\Sigma = \frac{f^2}{8}\mathrm{tr}[(D_\mu\Sigma)(D^\mu\Sigma)^\dagger]\,; \tag{2.2}$$

and L_{YK} the Yukawa couplings for fermions and scalars:

$$L_{YK} = -\frac{\lambda_1}{2}f\overline{u}_R\epsilon_{mn}\epsilon_{ijk}\Sigma_{im}\Sigma_{jn}\chi_{Lk} - \lambda_2 f\overline{U}_R U_L + \text{h.c.}\,. \tag{2.3}$$

In the above Lagrangians Σ is the GB matrix given by:

$$\Sigma = e^{2i\Pi/f}\Sigma_0 \tag{2.4}$$

where Σ_0 can be chosen to be:

$$\Sigma_0 = \begin{pmatrix} 0 & 0 & \mathbf{1} \\ 0 & 1 & 0 \\ \mathbf{1} & 0 & 0 \end{pmatrix}, \tag{2.5}$$

with $\mathbf{1}$ being the 2×2 unit matrix, and the Π matrix can be parameterized as:

$$\Pi \;=\; \begin{pmatrix} 0 & \frac{-i}{\sqrt{2}}H^\dagger & \phi^\dagger \\ \frac{i}{\sqrt{2}}H & 0 & \frac{-i}{\sqrt{2}}H^* \\ \phi & \frac{i}{\sqrt{2}}H^T & 0 \end{pmatrix}, \tag{2.6}$$

where $H = (H^0, H^+)$ is the SM Higgs doublet and ϕ is the triplet given by:

$$\phi = \begin{pmatrix} \phi^0 & \frac{1}{\sqrt{2}}\phi^+ \\ \frac{1}{\sqrt{2}}\phi^+ & \phi^{++} \end{pmatrix} . \tag{2.7}$$

The covariant derivative D_μ is defined as:

Model I

$$D_\mu \Sigma = \partial_\mu \Sigma - i \sum_{k=1}^{2} g_k W_k^a (Q_k^a \Sigma + \Sigma Q_k^{aT}) - i \sum_{k=1}^{2} g'_k B_k (Y_k \Sigma + \Sigma Y_k^T)$$

Model II

$$D_\mu \Sigma = \partial_\mu \Sigma - i \sum_{k=1}^{2} g_k W_k^a (Q_k^a \Sigma + \Sigma Q_k^{aT}) - i g' B (Y \Sigma + \Sigma Y^T) , \tag{2.8}$$

where g and g' are the gauge couplings, W_k^a $(a = 1, 2, 3)$ and B_k , B are the $SU(2)$ and $U(1)$ gauge fields respectively, $Q_{1ij}^a = \sigma_{ij}^a/2$ for $i, j = 1, 2$ and zero otherwise, $Q_{2ij}^a = \sigma_{i-3,j-3}^{a*}/2$ for $i, j = 4, 5$ and zero otherwise, $Y_1 = diag(-3, -3, 2, 2, 2)/10$, $Y_2 = diag(-2, -2, -2, 3, 3)/10$ and $Y = diag(-1, -1, 0, 1, 1)/2$. The Yukawa Lagrangian in (2.3) describes the interactions between GB and fermions, more exactly, the third generations of quarks plus the extra T quark appearing in the LH model. The indices in L_{YK} are defined such that $m, n = 4, 5$, $i, j = 1, 2, 3$, and

$$\begin{aligned} \overline{u}_R &= c\,\overline{t}_R + s\,\overline{T}_R , \\ \overline{U}_R &= -s\,\overline{t}_R + c\,\overline{T}_R, \end{aligned} \tag{2.9}$$

with:

$$\begin{aligned} c &= \cos\theta = \frac{\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}, \\ s &= \sin\theta = \frac{\lambda_1}{\sqrt{\lambda_1^2 + \lambda_2^2}} , \end{aligned} \tag{2.10}$$

and

$$\chi_L = \begin{pmatrix} u \\ b \\ U \end{pmatrix}_L = \begin{pmatrix} t \\ b \\ T \end{pmatrix}_L . \tag{2.11}$$

In addition to the above terms it is needed to add to the LH Lagrangian the Yang-Mills terms corresponding to the various gauge fields, and also the gauge fixing and Faddeev-Popov terms. Some of the gauge fields get massive at the tree level through the Higgs mechanism associated to the $SU(5)/SO(5)$ symmetry breaking. By using the Landau gauge, which is the most appropriate for the kind of computations we are presenting here (see [15] for further details), the quadratic part of the complete gauge boson Lagrangian can be written as:

$$L_\Omega = \frac{1}{2}\Omega^\mu((\Box + M_\Omega^2)g_{\mu\nu} - \partial_\mu\partial_\nu + 2\tilde{I}\, g_{\mu\nu})\Omega^\nu \,, \tag{2.12}$$

where Ω stands for any of the gauge bosons,

$$\begin{array}{ll} \textit{Model I} & \Omega^\mu = (W'^{\mu a}, W^{\mu a}, B'^\mu, B^\mu), \\ \textit{Model II} & \Omega^\mu = (W'^{\mu a}, W^{\mu a}, B^\mu)\,, \end{array} \tag{2.13}$$

being the mass matrix eigenstates,

$$\begin{array}{ll} \textit{Model I} & M_\Omega = (M_{W'} 1_{3\times3}, 0_{3\times3}, M_{B'}, 0), \\ \textit{Model II} & M_\Omega = (M_{W'} 1_{3\times3}, 0_{3\times3}, 0)\,, \end{array} \tag{2.14}$$

with $M_{W'} = f\sqrt{g_1^2 + g_2^2}/2$ and $M_{B'} = f\sqrt{g_1'^2 + g_2'^2}/\sqrt{20}$. The gauge boson mass eigenstates are defined such as:

$$\begin{aligned} W^a &= c_\psi W_1^a + s_\psi W_2^a, \\ W'^a &= s_\psi W_1^a - c_\psi W_2^a, \end{aligned} \tag{2.15}$$

where

$$\begin{aligned} s_\psi &= \sin\psi = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}, \\ c_\psi &= \cos\psi = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}, \end{aligned} \tag{2.16}$$

and

$$\begin{aligned} B &= c'_\psi B_1 + s'_\psi B_2, \\ B' &= s'_\psi B_1 - c'_\psi B_2, \end{aligned} \tag{2.17}$$

with

$$\begin{aligned} s'_\psi = \sin\psi' &= \frac{g'_1}{\sqrt{g'^2_1 + g'^2_2}}, \\ c'_\psi = \cos\psi' &= \frac{g'_2}{\sqrt{g'^2_1 + g'^2_2}}\,. \end{aligned} \tag{2.18}$$

$\tilde{I}$ is the interaction matrix between the gauge bosons and the H and ϕ scalars which can be found in our previous works [15, 16].

By adding the appropriate kinetic terms, the complete Lagrangian for the quarks becomes:

$$L_\chi = \overline{\chi}_R(i\partial\!\!\!/ - M + \hat{I})\chi_L + \text{h.c.}\,, \tag{2.19}$$

where

$$\chi_R = \begin{pmatrix} t \\ b \\ T \end{pmatrix}_R ,$$

$M = \text{diag}\,(0, 0, m_T)$ with $m_T = f\sqrt{\lambda_1^2 + \lambda_2^2}$ and $\hat{I}$ is the scalar-quark interaction matrix. The elements of this matrix can be found in [14, 16]. For more details about the model, including Feynman rules and also some phenomenological results see for example [12].

3 Effective operators

It is well known that the effective Higgs potential receive also contributions from additional operators coming from the ultraviolet completion of the LH model. Obviously these operators must be consistent with the symmetries of the LH model [2, 12, 18]. At the lowest order they can be parameterized by two unknown coefficients a and $a' \sim O(1)$. The form of these effective operators is, for the fermion sector [12]:

$$O_f = -a'\frac{1}{4}\lambda_1^2 f^4 \epsilon^{wx}\epsilon_{yz}\epsilon^{ijk}\epsilon_{kmn}\Sigma_{iw}\Sigma_{jx}\Sigma^{*my}\Sigma^{*nz}\,, \tag{3.1}$$

where i, j, k, m, n run over 1,2,3 and w, x, y, z run over 4,5 and for the gauge sector we have for *Model I*:

$$O_{gb} = \frac{1}{2}af^4\left\{g_j^2\sum_{a=1}^{3}\text{Tr}\left[(Q_j^a\Sigma)(Q_j^a\Sigma)^*\right] + g_j^{'2}\text{Tr}\left[(Y_j\Sigma)(Y_j\Sigma)^*\right]\right\}, \tag{3.2}$$

with $j = 1, 2$ and Q_j^a and Y_j being the generators of the $SU(2)_j$ and $U(1)_j$ groups, respectively. In the case of *Model II*:

$$O_{gb} = \frac{1}{2}cf^4\left\{g_j^2\sum_{a=1}^{3}\text{Tr}\left[(Q_j^a\Sigma)(Q_j^a\Sigma)^*\right] + g^{'2}\text{Tr}\left[(Y\Sigma)(Y\Sigma)^*\right]\right\}, \tag{3.3}$$

where $j = 1, 2$ and Y is the generator of the unique $U(1)$ group.

By expanding the GB field matrix Σ in these effective operators, we obtain their different contributions to the coefficients of the effective potential (1.2). The results are presented in the Appendix.

The complete result for the coefficients of the Higgs potential is given by the sum of the contributions coming from the effective operators, as given above, and the radiative contributions coming from all sectors of the model, as will be discussed in the following.

4 SSB and mass eigenstates

In the LH model the electroweak symmetry breaking is triggered, in principle, by the Higgs potential generated by one-loop radiative corrections, including both, fermion and gauge boson loops, and the effective operators introduced in the previous section. Obviously, this potential is invariant under the electroweak gauge group $SU(2) \times U(1)$ and also should have the correct form to break this symmetry spontaneously to $U(1)_{em}$. The relevant terms for this work are given in (1.2). Quartic terms involving ϕ^4 and $H^2\phi^2$ are not included since they give subleading contributions to the Higgs mass. These parameters were computed in our previous works [14–16] and are given in the Appendix for completeness.

The scalar potential, as given in (1.2), reaches its minimum at: $\langle HH^\dagger \rangle = v^2/2$ and $\langle \phi\phi^\dagger \rangle = v'^2$ with:

$$v^2 = \frac{\mu^2_{fg}}{\lambda_{fg} - \lambda^2_{H^2\phi}/\lambda_{\phi^2}}, \qquad v'^2 = \frac{\lambda_{H^2\phi}}{\sqrt{2}\lambda_{\phi^2}} \frac{v^2}{f}. \tag{4.1}$$

Note that both, the doublet and triplet scalars, get a *v.e.v.*, v and v' respectively. A standard choice for the components of these fields at the vacuum is:

$$H^+ = 0, \qquad H_0 = \frac{v}{\sqrt{2}}, \qquad \phi_0 = -v', \qquad \phi^+ = \phi^{++} = 0. \tag{4.2}$$

Then H and ϕ can be parameterized as:

$$H = (w^+, \frac{1}{\sqrt{2}}(v + h + iw_0)) \;\text{ and }\; \phi = \begin{pmatrix} -v' + \frac{1}{\sqrt{2}}(\xi + i\rho) & \frac{1}{\sqrt{2}}\phi^+ \\ \frac{1}{\sqrt{2}}\phi^+ & \phi^{++} \end{pmatrix}. \tag{4.3}$$

Obviously the new fields describe fluctuations around the vacuum and the potential written in terms of them can be split in four sectors, namely, the scalar, the pseudoscalar, the charged and the doubly charged. For the first three sectors we find that the new fields are not mass eigenstates. By diagonalizing the corresponding mass matrices we obtain the mass eigenstates in each case. I.e., for the scalar sector:

$$\begin{aligned} h &= c_0\mathcal{H} + s_0\Phi_0, & m_{\mathcal{H}}^2 &\equiv m_{fg}^2 = 2\,\mu_{fg}^2, \\ \xi &= c_0\Phi_0 - s_0\mathcal{H}, & m_{\Phi_0}^2 &= M_\phi^2 + 2\,m^2, \end{aligned} \tag{4.4}$$

the pseudoscalar sector:

$$\begin{aligned} w_0 &= c_P G^0 + s_P \Phi^P, & m_{G^0}^2 &= 0, \\ \rho &= c_P \Phi^P - s_P G^0, & m_{\Phi^P}^2 &= M_\phi^2 + 2m^2, \end{aligned} \tag{4.5}$$

and the charged sector:

$$\begin{aligned} w^+ &= c_+ G^+ + s_+ \Phi^+, & m_{G^+}^2 &= 0, \\ \phi^+ &= c_+ \Phi^+ + s_+ G^+, & m_{\Phi^+}^2 &= M_\phi^2 + m^2, \end{aligned} \tag{4.6}$$

with $M_\phi^2 = \lambda_{\phi^2} f^2$, $m^2 = v^2\lambda_{H^2\phi}^2/\lambda_{\phi^2}$. The doubly charged sector remains unchanged with a mass M_ϕ.

Where the notation introduced for the mass eigenstates is the following: $\mathcal{H}$ and Φ_0 are neutral scalars, Φ^P is a neutral pseudoscalar, Φ^+ and Φ^{++} are the charged and doubly charged scalars, and G^+ and G^0 are the would-be Goldstone bosons corresponding to the SM W and Z.

In terms of the mass eigenstates the leading order in the $\mathcal{O}(v^2/f^2)$ expansion of the potential is given by:

$$\begin{aligned} V_{eff} &= \frac{1}{2}m_{fg}^2\mathcal{H}^2 + \frac{1}{2}m_{\Phi_0}^2\Phi_0^2 + \frac{1}{2}m_{\Phi^P}^2{\Phi^P}^2 \\ &+ m_{\Phi^+}^2\Phi^+\Phi^- + v\lambda_{fg}\mathcal{H}^3 + v\lambda_{fg}{G^0}^2\mathcal{H} + 2v\lambda_{fg}G^+G^-\mathcal{H} \\ &+ \frac{\lambda_{fg}}{4}\mathcal{H}^4 + \frac{\lambda_{fg}}{2}\mathcal{H}^2{G^0}^2 + \lambda_{fg}\mathcal{H}^2G^+G^- \\ &- \frac{\lambda_{H^2\phi}}{\sqrt{2}}f\mathcal{H}^2\Phi_0 - \sqrt{2}\lambda_{H^2\phi}f\mathcal{H}G^0\Phi^P - \lambda_{H^2\phi}f(\mathcal{H}G^-\Phi^+ + \mathcal{H}G^+\Phi^-) + ... \end{aligned} \tag{4.7}$$

5 Goldstone boson sector contributions

The objective of this section is the computation of the radiative contributions to the Higgs mass and the Higgs quartic coupling coming from the GB sector.

The relevant Lagrangian is given by:

$$L_{GB} \;=\; \frac{1}{2}(\partial_\mu \Pi)(\partial^\mu \Pi) + \frac{1}{f^2}\left((\partial_\mu \Pi)(\partial^\mu \Pi)\Pi\Pi + \Pi(\partial_\mu \Pi)\Pi(\partial^\mu \Pi)\right) - V_{eff}. \tag{5.1}$$

In order to calculate the radiative contributions we write this Lagrangian in terms of the mass eigenstates and we split the Higgs field as $\mathcal{H} = \overline{\mathcal{H}} + \tilde{\mathcal{H}}$ where $\overline{\mathcal{H}}$ is the vacuum field and $\tilde{\mathcal{H}}$ describes the field fluctuations around this point. Then the first two terms of the Lagrangian above become:

$$\begin{aligned}
L_{Kin} \;=\;& \frac{1}{2}\left(1 + 2\frac{\overline{\mathcal{H}}^2}{f^2}\right)(\partial_\mu \tilde{\mathcal{H}})(\partial^\mu \tilde{\mathcal{H}}) + \frac{1}{2}\left(1 + \frac{\overline{\mathcal{H}}^2}{2f^2}\right)(\partial_\mu \Phi_0)(\partial^\mu \Phi_0) \\
+\;& \frac{1}{2}\left(1 + \frac{\overline{\mathcal{H}}^2}{2f^2}\right)(\partial_\mu G^0)(\partial^\mu G^0) + \frac{1}{2}\left(1 + \frac{\overline{\mathcal{H}}^2}{2f^2}\right)(\partial_\mu \Phi^P)(\partial^\mu \Phi^P) \\
+\;& \left(1 + \frac{\overline{\mathcal{H}}^2}{4f^2}\right)(\partial_\mu \Phi^+)(\partial^\mu \Phi^-) + \left(1 + \frac{\overline{\mathcal{H}}^2}{2f^2}\right)(\partial_\mu G^+)(\partial^\mu G^-) \\
+\;& (\partial_\mu \Phi^{++})(\partial^\mu \Phi^{--}).
\end{aligned} \tag{5.2}$$

Obviously, all the kinetic terms in this formula, but the last one, are not properly normalized. Therefore we write the fields in terms of a new set of properly normalized fields up to order $1/f^2$ as:

$$\Upsilon \;=\; \left(1 - \frac{\overline{\mathcal{H}}^2}{4f^2}\right)\Upsilon' \qquad \text{with} \qquad \Upsilon^{(')} = G^{0(')}, G^{\pm(')}, \Phi_0^{(')}, \Phi^{P(')}, \tag{5.3}$$

$$\tilde{\mathcal{H}} \;=\; \left(1 - \frac{\overline{\mathcal{H}}^2}{f^2}\right)\mathcal{H}', \tag{5.4}$$

$$\Phi^{\pm} \;=\; \left(1 - \frac{\overline{\mathcal{H}}^2}{8f^2}\right)\Phi^{\pm'}, \tag{5.5}$$

so that the Lagrangian is just:

$$\begin{aligned}
L_{Kin} \;=\;& \frac{1}{2}\partial_\mu \mathcal{H}'\partial^\mu \mathcal{H}' + \frac{1}{2}\partial_\mu \Phi_0'\partial^\mu \Phi_0' + \frac{1}{2}\partial_\mu G^{0'}\partial^\mu G^{0'} + \frac{1}{2}\partial_\mu \Phi^{P'}\partial^\mu \Phi^{P'} \\
+\;& \partial_\mu G^{+'}\partial^\mu G^{-'} + \partial_\mu \Phi^{+'}\partial^\mu \Phi^{-'} + \partial_\mu \Phi^{++}\partial^\mu \Phi^{--},
\end{aligned} \tag{5.6}$$

Then, the effective potential V_{eff} is given by:

$$V_{eff} = \frac{1}{2}m_{fg}^2\overline{\mathcal{H}}^2 + \frac{\lambda_{fg}}{4}\overline{\mathcal{H}}^4 + V_{eff}^{ss} + V_{eff}^{ps} + V_{eff}^{cs} + ... \tag{5.7}$$

where

$$
\begin{aligned}
V_{eff}^{ss} &= \frac{1}{2}m_{\Phi_0}^2\Phi_0^{'2} + \frac{1}{2}m_{fg}^2\mathcal{H}^{'2} + \frac{3}{2}\lambda_{fg}\overline{\mathcal{H}}^2\mathcal{H}^{'2} - \frac{\lambda_{\phi^2}}{4}\overline{\mathcal{H}}^2\Phi_0^{'2} \\
&- \sqrt{2}\lambda_{H^2\phi}f\overline{\mathcal{H}}\mathcal{H}'\Phi_0' - \frac{\lambda_{H^2\phi}}{\sqrt{2}}f\overline{\mathcal{H}}^{2'}\Phi_0', \qquad (5.8)\\
V_{eff}^{ps} &= \frac{1}{2}m_{\Phi^p}^2\Phi^{P'2} + \frac{\lambda_{fg}}{2}\overline{\mathcal{H}}^2 G^{0'2} - \frac{\lambda_{\phi^2}}{4}\overline{\mathcal{H}}^2\Phi^{P'2} \\
&- \sqrt{2}\lambda_{H^2\phi}f\overline{\mathcal{H}}G^{0'}\Phi^{P'}, \qquad (5.9)\\
V_{eff}^{cs} &= m_{\Phi^+}^2\Phi^{'+}\Phi^{'-} + \lambda_{fg}\overline{\mathcal{H}}^2 G^{'+}G^{'-} - \frac{\lambda_{\phi^2}}{4}\overline{\mathcal{H}}^2\Phi^{'+}\Phi^{'-} \\
&- \lambda_{H^2\phi}f\overline{\mathcal{H}}(G^{'-}\Phi^{'+} + G^{'+}\Phi^{'-}), \qquad (5.10)
\end{aligned}
$$

Observe that the third terms in (5.8), (5.9) and (5.10) describe the new interactions which come from the new normalization of the fields and the fact that the triplet boson mass is $\mathcal{O}(f^2)$. These interactions play a decisive role to cancel the quadratic divergences that come from the GB loops.

Finally, we can see that the split into different scalar sectors is maintained after diagonalization and normalization. This fact is important in order to simplify the computation of the radiative contributions coming from the GB. Thus we can deal with each scalar sector in an independent way being the computations in all cases similar. We illustrate this by computing the $(\mathcal{H}', \Phi_0')$ contribution and then we apply the same method to the other scalars.

5.1 Scalar sector contribution

The Lagrangian for the scalar sector $(\mathcal{H}', \Phi_0')$ is given by:

$$
\begin{aligned}
\mathcal{L}^{ss}(\overline{\mathcal{H}}, \mathcal{H}', \Phi_0') &= \frac{1}{2}\partial_\mu\mathcal{H}'\partial^\mu\mathcal{H}' + \frac{1}{2}\partial_\mu\Phi_0'\partial^\mu\Phi_0' - V_{eff}^{ss} \\
&= \frac{1}{2}\partial_\mu\mathcal{H}'\partial^\mu\mathcal{H}' + \frac{1}{2}\partial_\mu\Phi_0'\partial^\mu\Phi_0' - \frac{1}{2}m_{fg}^2\mathcal{H}^{'2} - \frac{1}{2}m_{\Phi_0}^2\Phi_0^{'2} \\
&- \frac{3}{2}\lambda_{fg}\overline{\mathcal{H}}^2\mathcal{H}^{'2} + \frac{\lambda_{\phi^2}}{4}\overline{\mathcal{H}}^2\Phi_0^{'2} + \sqrt{2}f\lambda_{H^2\phi}\overline{\mathcal{H}}\mathcal{H}'\Phi_0' + \frac{\lambda_{H^2\phi}}{\sqrt{2}}\overline{\mathcal{H}}^2\Phi_0'.
\end{aligned}
\qquad (5.11)
$$

The effective action for the $\overline{\mathcal{H}}$ is:

$$
e^{iS_{eff}[\overline{\mathcal{H}}]} = \int [d\mathcal{H}'][d\Phi_0']e^{i\int dx\mathcal{L}^{ss}}, \qquad (5.12)
$$

From the (5.11) we observe that the integration can be computed in two steps: First we concentrate on the Φ_0' field and then we integrate the $\mathcal{H}'$ field. After integrating Φ_0' we get the $\mathcal{H}$ effective action:

$$
\begin{aligned}
S^{ss}_{eff}[\overline{\mathcal{H}},\mathcal{H}'] &= -\frac{i}{2}\mathrm{Tr}\log\left[1+G_{\Phi_0}\frac{\lambda_{\phi^2}}{2}\overline{\mathcal{H}}^2\right] \\
&- f^2\lambda^2_{H^2\phi}\int dxdy\overline{\mathcal{H}}^2\mathcal{H}'_x G_{\Phi_0 xy}\mathcal{H}'_y - \frac{\lambda^2_{H^2\phi}}{4}f^2\int dxdy G_{\Phi_0 xy}\overline{\mathcal{H}}^4\delta_{yx} \\
&= -\frac{i}{2}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}\mathrm{Tr}\left(G_{\Phi_0}\frac{\lambda_{\phi^2}}{2}\overline{\mathcal{H}}^2\right)^k + \tilde{I}_2 + \tilde{I}_4\,,
\end{aligned} \tag{5.13}
$$

where the Φ_0' propagator is given by:

$$
G_{\Phi_0}(x,y) = \int d\tilde{k}e^{ik(x-y)}\frac{1}{k^2-m^2_{\Phi_0}}, \tag{5.14}
$$

here $\tilde{k}\equiv d^4k/(2\pi)^4$, and

$$
\tilde{I}_2 = -f^2\lambda^2_{H^2\phi}\int dxdy\overline{\mathcal{H}}^2\mathcal{H}'_x G_{\Phi_0 xy}\mathcal{H}'_y\,, \tag{5.15}
$$

$$
\tilde{I}_4 = -\frac{\lambda^2_{H^2\phi}}{4}f^2\int dxdy\overline{\mathcal{H}}^4\delta_{xy}G_{\Phi_0 xy}\,. \tag{5.16}
$$

Observe that we have obtained three terms. The first and the third ones are $\mathcal{H}'$ independent and they will give the Φ_0' radiative contributions to the Higgs mass and the quartic coupling.

Now integrating out $\mathcal{H}'$ we find its contribution to the $\overline{\mathcal{H}}$ effective action:

$$
\begin{aligned}
S^{ss}[\overline{\mathcal{H}}] &= -\frac{i}{2}\mathrm{Tr}\log\left[1+G_{\mathcal{H}'}(-3\lambda_{fg}\overline{\mathcal{H}}^2+2\tilde{I}_2)\right] \\
&= \frac{i}{2}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}\mathrm{Tr}\left(G_{\mathcal{H}}(3\lambda_{fg}\overline{\mathcal{H}}^2-2\tilde{I}_2)\right)+\dots\,,
\end{aligned} \tag{5.17}
$$

where $G_{\mathcal{H}'}$ is the $\mathcal{H}'$ propagator,

$$
G_{\mathcal{H}'}(x,y) = \int d\tilde{k}e^{ik(x-y)}\frac{1}{k^2-m^2_{fg}}\,. \tag{5.18}
$$

Finally, by taking into account (5.13) and (5.17), we obtain the $\overline{\mathcal{H}}$ effective action which reads:

$$\begin{aligned} S^{ss}[\overline{\mathcal{H}}] &= -\frac{i}{2}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}\mathrm{Tr}\left(G_{\Phi_0}\frac{\lambda_{\phi^2}}{2}\overline{\mathcal{H}}^2\right)^k \\ &\quad +\frac{i}{2}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}\mathrm{Tr}(G_{\mathcal{H}'}(3\lambda_{fg}\overline{\mathcal{H}}^2+\tilde{I}_2))^k+\tilde{I}_4. \end{aligned} \tag{5.19}$$

In order to obtain the scalar contribution to the Higgs mass we only need to consider the $k=1$ term in the expansion (5.19). The generic loop diagrams are shown in Fig. 1. Then, for $k=1$,

$$\begin{aligned} S^{(1)ss}[\overline{\mathcal{H}}] &= -\frac{i}{2}\lambda_{\phi^2}\int dxdy(G_{\Phi_0 xy}\overline{\mathcal{H}}^2\delta_{yx})+\frac{i}{2}\int dxdyG_{\mathcal{H}'xy}(3\lambda_{fg}\overline{\mathcal{H}}^2\delta_{yx}+\tilde{I}_2\delta_{yx}) \\ &= -\frac{i}{4}\lambda_{\phi^2}\int dx\overline{\mathcal{H}}^2 I_0(m^2_{\Phi_0})+\frac{3}{2}i\lambda_{fg}\int dx\overline{\mathcal{H}}^2 I_0(m^2_{fg}) \\ &\quad +i\lambda^2_{H^2\phi}f^2\int dx\overline{\mathcal{H}}^2 I_3(m^2_{\Phi_0},m^2_{fg})\,, \end{aligned} \tag{5.20}$$

with

$$I_0(M^2) \equiv \int d\tilde{k}\frac{i}{(k^2-M^2)}=\frac{1}{(4\pi)^2}\left[\Lambda^2-M^2\log\left(1+\frac{\Lambda^2}{M^2}\right)\right], \tag{5.21}$$

$$\begin{aligned} I_3(M_a^2,M_b^2) &\equiv \int d\tilde{p}\frac{i}{(p^2-M_a^2)(p^2-M_b^2)} \\ &= -\frac{1}{(4\pi)^2}\frac{1}{M_a^2-M_b^2}\left[M_a^2\log\left(1+\frac{\Lambda^2}{M_a^2}\right)-M_b^2\log\left(1+\frac{\Lambda^2}{M_b^2}\right)\right]. \end{aligned} \tag{5.22}$$

For the quartic coupling Higgs correction coming from $\tilde{I}_4$ we have:

$$\tilde{I}_4=\frac{\lambda^2_{H^2\phi}}{4\lambda_{\phi^2}}\int dx\overline{\mathcal{H}}^4+... \tag{5.23}$$

where we have expanded the Φ_0' propagator in powers of $k^2/m^2_{\Phi_0'}$ and kept just the first term.

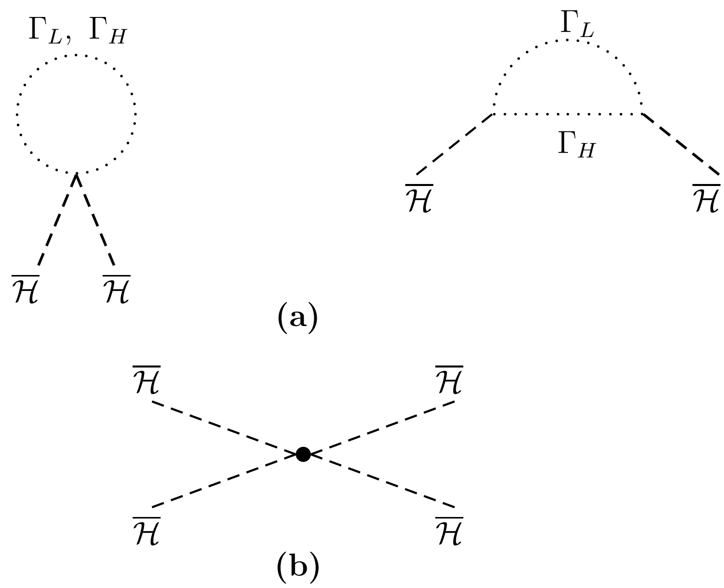

Figure 1: **(a)** Scalar sector loops contributing to the Higgs mass. $\Gamma_L = \mathcal{H}', G^{0'}$ or $G^{\pm'}$ and $\Gamma_H = \Phi_0', \Phi^P$ or $\Phi^{\pm'}$. **(b)** Contribution to the Higgs quartic coupling from the Φ_0' propagator.

5.2 Pseudoscalar sector and charged sector contributions

The computation of the contributions from the pseudoscalar and charged sectors is similar to the previous ones with only one difference, i.e.: these sectors do not give a contribution to the Higgs quartic coupling. They just contribute to the Higgs mass. Then the results for the pseudoscalar sector are:

$$\begin{aligned} S^{(1)ps}[\overline{\mathcal{H}}] &= -\frac{i}{4}\lambda_{\phi^2}\int dx\overline{\mathcal{H}}^2 I_0(m^2_{\Phi^p}) + \frac{1}{2}i\lambda_{fg}\int dx\overline{\mathcal{H}}^2 I_0(0) \\ &+ i\lambda^2_{H^2\phi}f^2\int dx\overline{\mathcal{H}}^2 I_3(m^2_{\Phi^p},0). \end{aligned} \tag{5.24}$$

and for the charged sector:

$$\begin{aligned} S^{(1)cs}[\overline{\mathcal{H}}] &= -\frac{i}{4}\lambda_{\phi^2}\int dx\overline{\mathcal{H}}^2 I_0(m^2_{\Phi^+}) + i\lambda_{fg}\int dx\overline{\mathcal{H}}^2 I_0(0) \\ &+ i\lambda^2_{H^2\phi}f^2\int dx\overline{\mathcal{H}}^2 I_3(m^2_{\Phi^+},0). \end{aligned} \tag{5.25}$$

Notice the there is no contribution coming from the doubly charged scalar sector.

5.3 Analytical results

Now by adding (5.20), (5.24), (5.25) we obtain the total radiative corrections to the Higgs mass from the GB sector up to order $\mathcal{O}(v^2/f^2)$ which reads:

$$\begin{aligned}\Delta m^2_{GB} &= \frac{3}{(4\pi)^2}\left\{\left(-\frac{\lambda_{\phi^2}}{4}+\lambda_{fg}\right)\Lambda^2+\left(\frac{\lambda_{\phi^2}}{4}+\frac{\lambda^2_{H^2\phi}}{\lambda_{\phi^2}}\right)M^2_\phi\log\left(1+\frac{\Lambda^2}{M^2_\phi}\right)\right.\\ &\left. -\frac{1}{2}\lambda_{fg}m^2_{fg}\log\left(1+\frac{\Lambda^2}{m^2_{fg}}\right)\right\}, \qquad (5.26)\end{aligned}$$

where, in order to simplify the computations, we have considered the heavy scalar fields as degenerate since m^2/M^2_ϕ is of the order of $\mathcal{O}(v^2/f^2)$ (see eq. (4.6)).

The coefficients of the Higgs potential $\lambda_{fg}, \lambda_{\phi^2}$ and $\lambda^2_{H^2\phi}$ appearing in eq. (5.26) receive contributions from both the radiative corrections and the effective operators (see Appendix). Since the contributions to λ_{fg} and λ_{ϕ^2} contain terms of the order of Λ^2, divergencies $\mathcal{O}(\Lambda^4)$ and $\mathcal{O}(\Lambda^2)$ emerge from the first term in (5.26). However, these divergencies cancel due to the relationship between λ_{ϕ^2} and λ_{fg}, namely:

$$\begin{aligned}\lambda^{\Lambda^2}_{fg} &= \frac{1}{4}\lambda^{\Lambda^2}_{\phi^2},\\ \lambda^{EO}_{fg} &= \frac{1}{4}\lambda^{EO}_{\phi^2}, \qquad (5.27)\end{aligned}$$

where the index Λ^2 refers to the quadratically divergent terms and EO represents the part of these coefficients coming from the effective operators. This fact occurs in the fermionic and gauge boson sectors, where the quadratic divergences coming from light and heavy modes of the same statistics cancel [2]. Then the corrections summarized in Δm^2_{GB} (eq. 5.26) are at most of the order $\mathcal{O}(\Lambda^2\log(\Lambda^2/M^2))$. It is important to stress that the above cancellations occur exactly only in *Model I* (as you can easily check from the results given in the Appendix). However, in *Model II* (where only the $SU(2)\times SU(2)\times U(1)$ is gauged), there are $\mathcal{O}(\Lambda^2)$ terms coming from the U(1) sector which do not cancel. However, such terms appear always with a squared gauge coupling g' factor which is very small ($g'^2/g^2\sim 0.3$ in the SM) and then their contribution is not expected to be too large.

Finally, from (5.23), the radiative correction to the quartic coupling is:

$$\tilde{I}_4 = \frac{1}{4}\Delta\lambda_{GB}\int dx\overline{\mathcal{H}}^4,$$

being

$$\Delta\lambda_{GB} = \frac{\lambda^2_{H^2\phi}}{\lambda_{\phi^2}}. \tag{5.28}$$

In summary, taking into account (5.23) and (5.26), the Higgs boson potential can be written as:

$$V = \frac{1}{2} m^2_{\overline{\mathcal{H}}} \overline{\mathcal{H}}^2 + \frac{1}{4} \lambda_{\overline{\mathcal{H}}} \overline{\mathcal{H}}^4, \tag{5.29}$$

where the Higgs mass is given by,

$$m^2_{\overline{\mathcal{H}}} = 2(\mu^2_{fg} - \Delta m^2_{GB}), \tag{5.30}$$

and the quartic Higgs couplings is,

$$\lambda_{\overline{\mathcal{H}}} = \lambda_{fg} - \frac{\lambda^2_{H^2\phi}}{\lambda_{\phi^2}}. \tag{5.31}$$

It is important to note that we have obtained the GB contributions after having broken the SM symmetry through the fermion and gauge boson radiative corrections. In this fact we differ from other analysis performed in the literature (see for example [2, 18]), where these scalar contributions are computed at the tree level from the effective operators only. Moreover, in our case, the coefficients of the potential (1.2) do not depend only on the two unknown coefficients a and a', but also on the scale f and the cutoff Λ, thus setting more restrictions on the space parameter as we will see in the following.

6 Numerical Results and Phenomenological Discussion

In this section we continue our study about the allowed region of the parameter space of the LH model started in our previous papers [14–16]. In the present one we complete this phenomenological study, taking into account also the contributions from the Goldstone boson sector to the Higgs mass and quartic coupling obtained above. The LH parameters different relationships and their relevant ranges considered are the following:

First, we impose the minimum condition for the complete effective potential (1.2):

$$v^2 = \frac{\mu^2_{fg}}{\lambda_{fg} - \lambda^2_{H^2\phi}/\lambda_{\phi^2}}\,. \tag{6.1}$$

This condition is crucial in order to reproduce the electroweak symmetry breaking.

If we want to study the allowed region of the parameter space in these models, we should also take into account other constraints imposed by requiring the consistency of the LH models with the electroweak precision data. There exist several studies of the corrections to electroweak precision observables in the Little Higgs models, exploring whether there are regions of the parameter space in which the model is consistent with the available data [4,5,7–13]. In *Model I* with a gauge group $SU(2)\times SU(2)\times U(1)\times U(1)$ we have a multiplet of heavy $SU(2)$ gauge bosons and a heavy $U(1)$ gauge boson. The last one leads to large electroweak corrections and some problems with the direct observational bounds on the Z' boson from Tevatron [7,8]. Then, a very strong bound on the symmetry breaking scale f, $f > 4$ TeV at 95% C.L, is found [7]. However, it is known that this bound is lowered to $1-2$ TeV for some region of the parameter space [8] by gauging only $SU(2)\times SU(2)\times U(1)$ (*Model II*). For this reason, in the following we will concentrate only on this model.

On the other hand, in order to avoid small values for the W' mass and a very strong coupling constant, we set the range of the ψ mixing angle (for the $SU(2)$ group) to be $0.1 < c_\psi < 0.9$ [15]. In addition, the condition $\lambda_T \gtrsim 0.5$ is established from the top mass [12], setting the bounds on the couplings $\lambda_1, \lambda_2 \geq m_t/v$ or $\lambda_1\lambda_2 \geq 2(m_t/v)^2$. In order to avoid a large fine-tuning in the Higgs potential [2,13] we set the condition $m_T \lesssim 2.5$ TeV. Then, since m_T grows linearly with f, f should be less than about one TeV [14]. Following the restrictions on the parameters given in [15], we take 0.8 TeV $< f < 1$ TeV. Finally the usual condition $\Lambda \lesssim 4\pi f$ is also imposed.

By using the constraints on the LH parameters given above, taking into account also that the Higgs mass is experimentally restricted to the range 114 GeV $< m_{\overline{\mathcal{H}}} < 200$ GeV, and by imposing the minimum condition (6.1), we analyze the available regions for the remaining LH parameters. To do that we include the contributions of both radiative corrections and effective operators. In fact, in order to see the role played for each of them, we consider three different cases: having just radiative corrections (RC), just effective operators (EO) and the most general case including both of them (RC+EO).

In Fig. 2 we show the allowed regions of the parameter space for the three different cases analyzed; RC (red region), EO (blue region) and RC+EO (green region). In Fig. 2.a we show the possible solutions to the LH model in the $(\Lambda, c_\psi, \lambda_T)$ space varying f between 0.8 TeV and 1 TeV and by

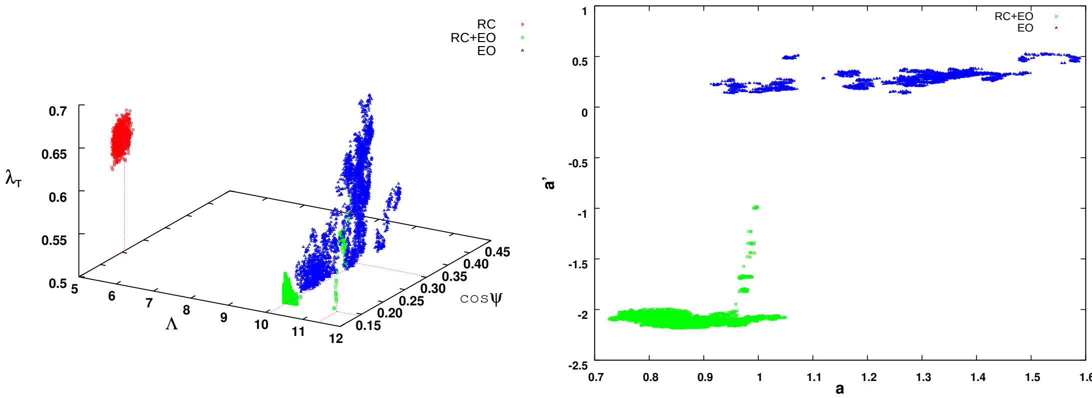

Figure 2: **(a)** Values of λ_T , Λ and c_ψ which are possible solutions for the LH model. Here f vary between 0.8 and 1 TeV, and a and a' are $\mathcal{O}(1)$. The three separate surfaces correspond with the three different cases analyzed in this section. **(b)** Values of a and a' which are possible solutions for the LH model. The λ_T , Λ , c_ψ and f ranges are described in the text.

assuming that the a and a' parameters are of the order of $\mathcal{O}(1)$. From these results there are two important issues to remark. First, when only radiative corrections are included we do not find any solution for the LH model if $\Lambda > 6$ TeV. Unfortunately, precision electroweak data rule out new strong interactions at scales below about 10 TeV. On the contrary, in the other two cases, RC+EO and EO, the possible values for the cut-off are larger. This fact implies also that the mass of the ϕ fields must be about 2 TeV when the model includes only radiative corrections unlike in the other two cases where it is about 5 TeV (see Fig. 3). In Fig. 2.b we show the possible values for the unknown a and a' parameters. Here, the other parameters have been varied in the ranges set above. The two cases considered are RC+EO and EO only. We find that the set of possible solutions include in both cases positive values for a . In the RC+EO case we obtain large and negative values for a' , whereas in the EO case a' takes small and positive values. Notice also that a is always positive. This is important since it is known that $a < 0$ leads to a large $v.e.v$ for the scalar triplet.

The reason for the differences of the parameter solutions for the three cases come from the Δm^2_{GB} cutoff dependence when the radiative contributions are included. For example, in the case where only the radiative corrections are taken into account, a cut-off Λ bigger than 6 TeV produces GB contributions resulting in a negative Higgs mass. However, by dropping the value of Λ we get a LH parameter space where the condition (6.1) is satisfied and the Higgs mass is well inside the experimental constraints. In the RC+EO case, the a' parameter can take values which help to compensate the big effect of the GB radiative contributions (see also Fig. 4) thus allowing larger cutoff values.

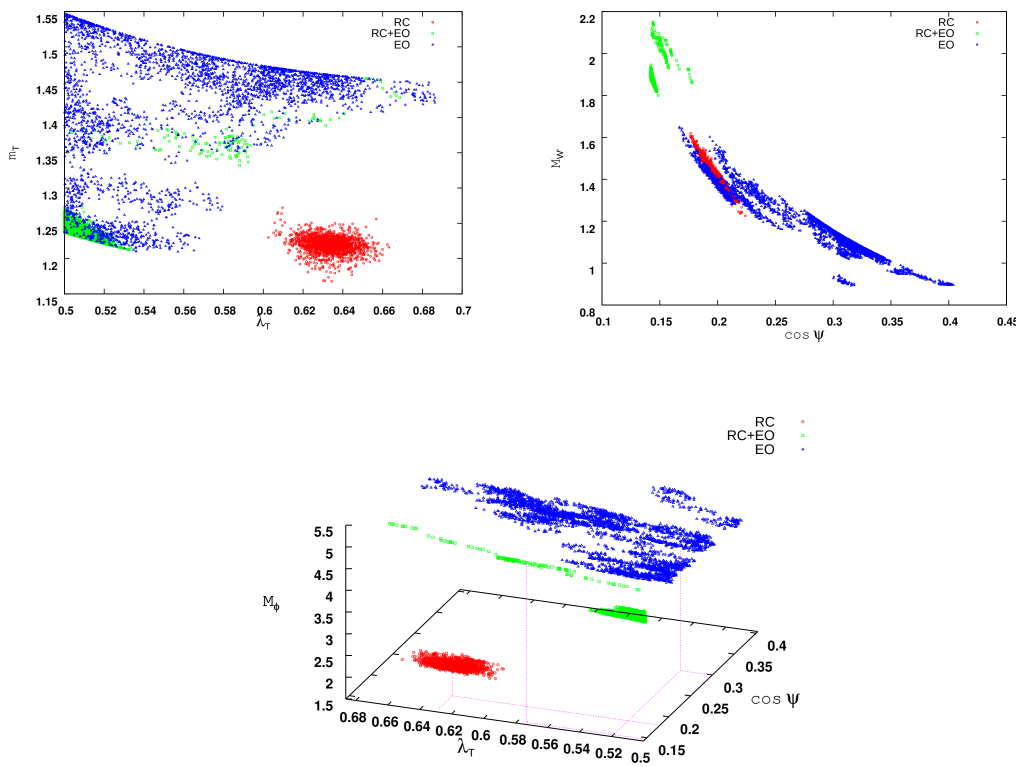

Figure 3: **(a)** m_T as a function of λ_T, **(b)** $M_{W'}$ as a function of $\cos\psi$ and **(c)** M_ϕ as a function of λ_T and $\cos\psi$, where the Λ, f, a and a' parameters vary between ranges described in the text.

For completeness, Fig. 3 shows the mass values for the heavy particles in the three different cases analyzed. Each point of the figures is a possible solution of the LH model. In this way, these regions represent the possible values for the masses of the heavy particles predicted by the LH model, which are compatible with electroweak symmetry breaking and precision data. The region of possible values for the masses coming from EO contributions is clearly larger than in the case of considering RC alone. Notice that the theoretical lower bounds in the heavy states masses, $M_\phi, M_{W'} \gtrsim 1$ TeV, and the condition $m_T \lesssim 2.5$ TeV are fulfilled.

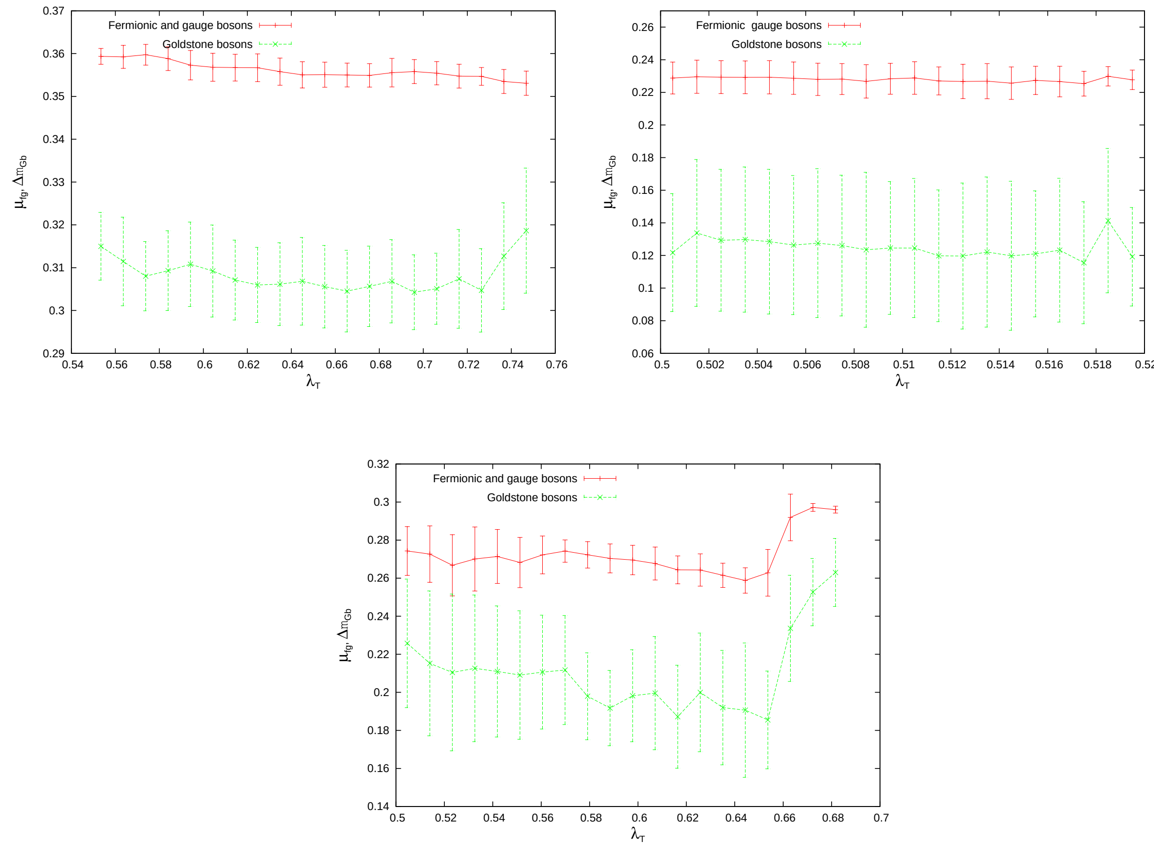

Figure 4: These figures show the average and standard deviation of both the fermionic and gauge boson contribution μ_{fg} and the Goldstone boson contribution Δm_{GB} to the Higgs mass as a function of λ_T. **(a)** RC case, **(b)** RC+EO case and **(c)** EO case.

To complete our study, we compare the contributions to the Higgs mass coming from the different sectors i.e. fermionic and gauge bosons (μ_{fg}) and on the other hand the GB contribution (Δm_{GB}), as a function of λ_T. We show the average and standard deviation for each contribution (Fig. 4). In all physical cases it can be seen that $\mu_{fg} > \Delta m_{GB}$, thus yielding a real value for the Higgs mass (eq. 5.30). It is also remarkable the higher variability of Δm_{GB} compared with μ_{fg}. The reason is that both the parameters appearing in the radiative corrections, i.e. $f, \Lambda, \lambda_T, \cos\phi$, and the two EO parameters a and a', play an important role in the final results of Δm_{GB} (see the discussion above).

Finally, as an example, we give in the Table.1 the lowest Higgs mass values found for the three cases considered in this work.

Parameters	RC	RC+EO	EO
$m_{\overline{\mathcal{H}}}$	156.66 GeV	114.69 GeV	116.94 GeV
μ_{fg}	359.54 GeV	236.87 GeV	288.70 GeV
Δ_{Gb}	342.04 GeV	222.55 GeV	275.53 GeV
$\lambda_{\overline{\mathcal{H}}}$	0.97	0.90	1.42
f	0.86 TeV	0.96 TeV	0.82 TeV
Λ	5 TeV	11.64 TeV	10.01 TeV
λ_T	0.6	0.61	0.53
c_ψ	0.18	0.16	0.3
a	0	0.98	1.06
a'	0	-1.25	0.5

Table 1: The lowest values for the Higgs mass found for the three cases: RC, RC+EO and EO.

7 Conclusion

In this work we have completed our program of computing the relevant contributions to the Higgs low-energy effective potential in the context of the Littlest Higgs models based on the $SU(5)/SO(5)$ coset. To the radiative corrections coming from the fermions and the gauge bosons considered so far, we have added here the effect the scalar loops and also the effective operators emerging from the ultraviolet completion of the model.

In particular we have computed in detail the main contributions to the Higgs mass and its quartic coupling. From our previous works, in which only fermionic and gauge boson radiative corrections were included, it was clear that the effect of the scalar sector could be decisive in order to have the appropriate cancellations between the different sectors of the model to give a Higgs mass within the present experimental limits. We have performed our analytical computations for two different versions of the model called *Model I* and *Model II* having as gauge groups $[SU(2)\times U(1)]^2$ and $SU(2)^2\times U(1)$ respectively.

In order to complete our analysis, we have concentrated on studying those regions of the parameter space where the model could give rise to an acceptable phenomenology. In particular we have done a detailed numerical search for *Model II* since *Model I* seems to be incompatible with the present experimental data [7, 8]. We have analyzed three cases: 1) radiative corrections only (RC), 2) radiative corrections and effective operators (RC+EO) and 3) effective operator only (EO). From this analysis we get that this model is compatible with the expected Higgs mass provided that the contribution of the effective operators is included. We also conclude that the Goldstone boson contributions are fundamental to obtain a low enough Higgs particle mass. For example a Higgs mass $m_H \simeq 115 GeV$ can be obtained when radiative and effective operator contributions are both taken into account.

Summarizing, we have arrived to the conclusion that the $SU(5)/S(5)$ Littlest Higgs model with gauge group $[SU(2) \times U(1)]^2$ is phenomenologically viable through some tuning in the parameter space, assuming a careful inclusion of fermions, gauge bosons, scalar loops and effective operators.

In any case it will be the LHC, whose main goal is to disentangle the mechanism of the electroweak symmetry breaking, which will decide if Littlest Higgs models are appropriate for describing mechanism or not.

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Appendix

a. Coefficients coming from loops computation

Model I

$$
\begin{aligned}
\mu_{fg}^2 &= \mu_f^2 + \mu_g^2 \\
&= N_c \frac{m_T^2 \lambda_t^2}{4\pi^2} \log\left(1 + \frac{\Lambda^2}{m_T^2}\right) \\
&- \frac{3}{64\pi^2}\left[3g^2 M_{W'}^2 \log\left(1 + \frac{\Lambda^2}{M_{W'}^2}\right) + g'^2 M_{B'}^2 \log\left(1 + \frac{\Lambda^2}{M_{B'}^2}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
\lambda_f &= \frac{N_c}{(4\pi)^2}\left[2(\lambda_t^2 + \lambda_T^2)\frac{\Lambda^2}{f^2}\right. \\
&- \log\left(1 + \frac{\Lambda^2}{m_T^2}\right)\left(-\frac{2m_T^2}{f^2}\left(\frac{5}{3}\lambda_t^2 + \lambda_T^2\right) + 4\lambda_t^4 + 4(\lambda_T^2 + \lambda_t^2)^2\right) \\
&- \left.4\lambda_T^2 \frac{1}{1 + \frac{m_T^2}{\Lambda^2}}\left(\frac{m_T^2}{f^2} - 2\lambda_t^2 - \lambda_T^2\right) - 4\lambda_t^4 \log\left(\frac{\Lambda^2}{m^2}\right)\right] \\
&- \frac{3}{(16\pi f)^2}\left[-\left(\frac{g^2}{c_\psi^2 s_\psi^2} + \frac{g'^2}{c_\psi'^2 s_\psi'^2}\right)\Lambda^2\right. \\
&+ g^2 M_{W'}^2 \log\left(1 + \frac{\Lambda^2}{M_{W'}^2}\right)\left(4 + \frac{1}{c_\psi^2 s_\psi^2} + 2g'^2 \frac{(c_\psi^2 s_\psi'^2 + s_\psi^2 c_\psi'^2)^2}{c_\psi^2 s_\psi^2 c_\psi'^2 s_\psi'^2}\frac{f^2}{M_{W'}^2 - M_{B'}^2}\right) \\
&+ g'^2 M_{B'}^2 \log\left(1 + \frac{\Lambda^2}{M_{B'}^2}\right)\left(\frac{4}{3} + \frac{1}{c_\psi'^2 s_\psi'^2} + 2g^2 \frac{(c_\psi^2 s_\psi'^2 + s_\psi^2 c_\psi'^2)^2}{c_\psi^2 s_\psi^2 c_\psi'^2 s_\psi'^2}\frac{f^2}{M_{B'}^2 - M_{W'}^2}\right) \\
&+ f^2 \log\left(1 + \frac{\Lambda^2}{M_{W'}^2}\right)\left(3g^4 + 2(3g^2 + g'^2)g^2 \frac{(s_\psi^2 - c_\psi^2)^2}{c_\psi^2 s_\psi^2}\right) \\
&+ f^2 \log\left(1 + \frac{\Lambda^2}{M_{B'}^2}\right)\left(g'^4 + 2(g^2 + g'^2)g'^2 \frac{(s_\psi'^2 - c_\psi'^2)^2}{c_\psi'^2 s_\psi'^2}\right) \\
&+ \left. f^2 \log\left(\frac{\Lambda^2}{m^2}\right)\left(3g^4 + g'^4 + 8g^2 g'^2\right) - 3f^2 \frac{g^4}{1 - \frac{M_{W'}^2}{\Lambda^2}} - f^2 \frac{g'^4}{1 - \frac{M_{B'}^2}{\Lambda^2}}\right]
\end{aligned}
$$

$$\begin{aligned}
\lambda_{\phi^2 f} &= \frac{8N_c}{(4\pi f)^2}(\lambda_t^2+\lambda_T^2)\left(\Lambda^2 - m_T^2\log\left(\frac{\Lambda^2}{m_T^2}+1\right)\right) \\
&+ \frac{3}{4(4\pi f)^2}\left[\frac{g^2}{c_\psi^2 s_\psi^2}\Lambda^2 - g^2 M_{W'}^2 \log\left(\frac{\Lambda^2}{M_{W'}^2}+1\right)\left(\frac{(s_\psi^2-c_\psi^2)^2}{c_\psi^2 s_\psi^2}-4\right)\right. \\
&+ \left.\frac{g'^2}{c_{\psi'}^2 s_{\psi'}^2}\Lambda^2 - g'^2 M_{B'}^2 \log\left(\frac{\Lambda^2}{M_{B'}^2}+1\right)\frac{(s_{\psi'}^2-c_{\psi'}^2)^2}{c_{\psi'}^2 s_{\psi'}^2}\right]
\end{aligned}$$

$$\begin{aligned}
\lambda_{H^2\phi} &= -\frac{4N_c}{(4\pi f)^2}\left[(\lambda_t^2+\lambda_T^2)\Lambda^2 - \lambda_T^2 m_T^2 \log\left(\frac{\Lambda^2}{m_T^2}+1\right)\right] \\
&+ \frac{3}{8(4\pi f)^2}\left[g^2\frac{s_\psi^2-c_\psi^2}{c_\psi^2 s_\psi^2}\left(\Lambda^2 - M_{W'}^2\log\left(\frac{\Lambda^2}{M_{W'}^2}+1\right)\right)\right. \\
&+ \left. g'^2\frac{s_{\psi'}^2-c_{\psi'}^2}{c_{\psi'}^2 s_{\psi'}^2}\left(\Lambda^2 - M_{B'}^2\log\left(\frac{\Lambda^2}{M_{B'}^2}+1\right)\right)\right],
\end{aligned}$$

Model II

$$\begin{aligned}
\mu_{fg}^2 &= \mu_f^2+\mu_g^2 \\
&= N_c\frac{m_T^2\lambda_t^2}{4\pi^2}\log\left(1+\frac{\Lambda^2}{m_T^2}\right) - \frac{3}{64\pi^2}\left(3g^2 M_{W'}^2\log\left(1+\frac{\Lambda^2}{M_{W'}^2}\right)+g'^2\Lambda^2\right)
\end{aligned}$$

$$\begin{aligned}
\lambda_{fg} &= \frac{N_c}{(4\pi)^2}\left[2(\lambda_t^2+\lambda_T^2)\frac{\Lambda^2}{f^2}\right. \\
&- \log\left(1+\frac{\Lambda^2}{m_T^2}\right)\left(-\frac{2m_T^2}{f^2}\left(\frac{5}{3}\lambda_t^2+\lambda_T^2\right)+4\lambda_t^4+4(\lambda_T^2+\lambda_t^2)^2\right) \\
&- \left. 4\lambda_T^2\frac{1}{1+\frac{m_T^2}{\Lambda^2}}\left(\frac{m_T^2}{f^2}-2\lambda_t^2-\lambda_T^2\right)-4\lambda_t^4\log\left(\frac{\Lambda^2}{m^2}\right)\right] \\
&- \frac{3}{(16\pi f)^2}\left[-\frac{g^2}{c_\psi^2 s_\psi^2}\Lambda^2+\frac{4}{3}g'^2\Lambda^2+g^2M_{W'}^2\log\left(\frac{\Lambda^2}{M_{W'}^2}+1\right)\left(4+\frac{1}{c_\psi^2 s_\psi^2}\right)\right. \\
&+ f^2\log\left(1+\frac{\Lambda^2}{M_{W'}^2}\right)\left(3g^4+2(3g^2+g'^2)g^2\frac{(s_\psi^2-c_\psi^2)^2}{s_\psi^2 c_\psi^2}\right) \\
&+ \left. f^2\log\left(\frac{\Lambda^2}{m^2}\right)(3g^4+g'^4+8g^2g'^2)-3f^2\frac{g^4}{1-\frac{M_{W'}^2}{\Lambda^2}}\right]
\end{aligned}$$

$$
\begin{aligned}
\lambda_{\phi^2} &= \frac{8N_c}{(4\pi f)^2}(\lambda_t^2+\lambda_T^2)\left(\Lambda^2 - m_T^2 \log\left(\frac{\Lambda^2}{m_T^2}+1\right)\right)\\
&+ \frac{3}{64\pi^2 f^2}\left[\frac{g^2}{c_\psi^2 s_\psi^2}\Lambda^2 - g^2 M_{W'}^2 \log\left(\frac{\Lambda^2}{M_{W'}^2}+1\right)\left(\frac{(s_\psi^2-c_\phi^2)^2}{c_\psi^2 s_\psi^2}-4\right)\right]\\
&+ \frac{3g'^2}{(4\pi f)^2}\Lambda^2
\end{aligned}
$$

$$
\begin{aligned}
\lambda_{H^2\phi} &= -\frac{4N_c}{(4\pi f)^2}\left[(\lambda_t^2+\lambda_T^2)\Lambda^2 - \lambda_T^2 m_T^2 \log\left(\frac{\Lambda^2}{m_T^2}+1\right)\right]\\
&+ \frac{3g^2}{8(4f\pi)^2}\frac{s_\psi^2-c_\psi^2}{c_\psi^2 s_\psi^2}\left(\Lambda^2 - M_{W'}^2 \log\left(\frac{\Lambda^2}{M_{W'}^2}+1\right)\right)\,,
\end{aligned}
$$

b. Coefficients coming from effective operators

Modelo I

$$
\begin{aligned}
\lambda_{fg}^{\mathrm{EO}} &= \frac{a}{8}\left(\frac{g^2}{s_\psi^2 c_\psi^2}+\frac{g'^2}{s_\psi'^2 c_\psi'^2}\right)+2a'(\lambda_t^2+\lambda_T^2)\\
\lambda_{\phi^2}{}^{\mathrm{EO}} &= \frac{a}{2}\left(\frac{g^2}{s_\psi^2 c_\psi^2}+\frac{g'^2}{s_\psi'^2 c_\psi'^2}\right)+8a'(\lambda_t^2+\lambda_T^2)\\
\lambda_{H^2\phi}{}^{\mathrm{EO}} &= \frac{a}{4}\left(g^2\frac{c_\psi^2-s_\psi^2}{s_\psi^2 c_\psi^2}+g'^2\frac{c_\psi'^2-s_\psi'^2}{s_\psi'^2 c_\psi'^2}\right)+4a'(\lambda_t^2+\lambda_T^2)
\end{aligned}
$$

Modelo II

$$
\begin{aligned}
\lambda_{fg}^{\mathrm{EO}} &= \frac{a}{8}\left(\frac{g^2}{s_\psi^2 c_\psi^2}\right)-\frac{a}{3}g'^2+2a'(\lambda_t^2+\lambda_T^2)\\
\lambda_{\phi^2}{}^{\mathrm{EO}} &= \frac{a}{2}\left(\frac{g^2}{s_\psi^2 c_\psi^2}\right)+4ag'^2+8a'(\lambda_t^2+\lambda_T^2)\\
\lambda_{H^2\phi}{}^{\mathrm{EO}} &= \frac{a}{4}g^2\frac{c_\psi^2-s_\psi^2}{s_\psi^2 c_\psi^2}+4a'(\lambda_t^2+\lambda_T^2)\\
\mu^{2\,\mathrm{EO}} &= af^2g'^2
\end{aligned}
$$

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